Moragaspitiya, Praveen, Thambiratnam, David, Perera, Nimal, & Chan, Tommy H.T. (2011) Quantifying axial deformation of columns using vibration characteristics. In *The First International Postgraduate Conference on Engineering, Designing and Developing the Built Environment for Sustainable Wellbeing*, 27-29 April 2011, Queensland University of Technology, Brisbane, Qld.

QUANTIFYING AXIAL DEFORMATIONS OF COLUMNS USING VIBRATION CHARACTERISTICS

MORAGASPITIYA, H.N.P. *†, THAMBIRATNAM, D.P. *, PERERA, N.J. *‡, & CHAN, T.H.T. * pp. 225-230

* Faculty of Built Environment and Engineering, Queensland University of Technology, Brisbane, Australia

† E-mail: praveenqut@gmail.com

‡ Group Technical Consultant, Robert Bird Group, Brisbane, Australia

Abstract: Column elements at a certain level in building are subjected to loads from different tributary areas. Consequently, differential axial deformation among these elements occurs. Adverse effects of differential axial deformation increase with building height and geometric complexity. Vibrating wire, electronic strain and external mechanical strain gauges are used to measure the axial deformations to take adequate provisions to mitigate the adverse effects. These gauges require deploying in or on the elements during their construction in order to acquire necessary measurements continuously. The use of these gauges is therefore inconvenient and uneconomical. This highlights the need for a method to quantify the axial deformation using ambient measurements. This paper proposes a comprehensive vibration based method. The unique capabilities of the proposed method present through an illustrative example.

Key words: Axial Deformation. Column Elements Vibration Characteristics, Dynamic Stiffness Matrix, Finite Element Technique.

1 INTRODUCTION

Planning is underway worldwide for the construction of geometrically complex high rise buildings. These buildings comprise column elements to carry gravity load. Fig. 01 shows a geometrically complex high rise building, “Lagoon” proposed for Dubai. Column elements at a certain level in buildings are subjected to loads from different tributary areas leading to differential axial deformation which enhances with building height and geometric complexity. Gauges such as vibrating wire, electronic strain and external mechanical strain gauges are used to measure the axial deformations to take adequate provisions to mitigate the adverse effects of the differential axial deformation. However, these gauges need to be installed in or on the elements during their constructions to acquire measurements continuously. Consequently, use of these gauges; protecting and continuous data acquisition is inconvenient and uneconomical (Carreira & Poulos, 2007). This paper thereby presents a comprehensive vibration based method to quantify axial deformations of column elements.

FIGURE 01: The Lagoons - proposed for Dubai (Dubai Future Projects, 2009)

Measuring vibrations characteristics of buildings is becoming increasingly popular due to the fact that they can be conveniently used to assess building health and performance. An ambient vibration test on the Republic Plaza, one of the tallest buildings in Singapore, was conducted over a two year period, from the commencement of construction to the service stages, to assess the performance during these stages (Brownjohn, Pan & Deng, 2009). Ellis & Ji (1996) studied the dynamic characteristics of a building during and after its construction. The study was conducted using a reasonably large building model to understand the behaviour of structural components under dynamic excitations such as wind, traffic loads, earthquakes etc. These two studies confirmed the validity of using vibration characteristics to evaluate building behaviour.

2 METHODOLOGY

Influence of axial deformations of column elements in building structural framing systems on the vibration characteristics can be investigated through the dynamic stiffness matrix. Section 2.1 will present development of the dynamic stiffness matrix. A Modal Flexibility phenomenon (MF) comprising modal vectors and natural frequencies, is used to develop the relationship between axial deformation and vibration characteristics. Section 2.2 present more details of MF phenomenon.

2.1 Dynamic Stiffness Matrix

A beam element with a fixed end condition subjected to vibration with axial compressive force is shown in Fig. 02.

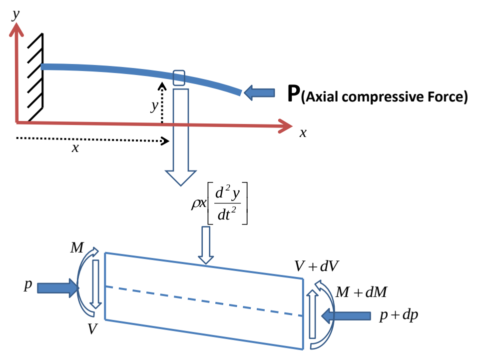


FIGURE 02: An element with axial compressive force under free vibration

In Fig. 02, where P- axial compressive force, M-moment, V-shear force, ρ-mass per unit length, y, x-distances considered, t-time

Equation (1) can be formed using moments and forces of the element depicted in Fig. 02:

$$EI\left[\frac{d^4y(x,t)}{dx^4}\right]+P\left[\frac{d^2y(x,t)}{dx^2}\right]+\rho\left[\frac{d^2y(x,t)}{dt^2}\right]=0 \qquad (1)$$

Equation (1) is a fourth order homogeneous differential equation and can be solved using variable separation method. y(x,t) can thus be written using two parameters such as Z(x) and w(t) to represent the influence of deflection and time respectively. Solutions of (1) can thereby written as:

$$y(x,t) = Z(x)w(t) \tag{2}$$

where,

$$z(x) = D_1 \sinh(\alpha_1 x) + D_2 \cosh(\alpha_1 x) + D_3 \sin(\alpha_2 x) + D_4 \cos(\alpha_2 x)$$

$$w(t) = A_1 \sin(\beta_1 t) + A_2 \cos(\beta_2 t)$$

D_1, D_2, D_3, D_4- constants.

A_1, A_2- constants determined from initial conditions of the vibration.

$$\alpha_2^2 = \frac{-P - \sqrt{p^2 + 4EI\omega^2}}{2EI}$$

$$\alpha_1^2 = \frac{-P + \sqrt{p^2 + 4EI\omega^2}}{2EI}$$

$$\beta_1 = \sqrt{\frac{\omega^2}{\rho}}$$

$$\beta_2 = -\sqrt{\frac{\omega^2}{\rho}}$$

Equation (3) can be formed as follows considering boundary conditions:

$$[Y] = [A][D] \tag{3}$$

where,

$$[y] = \begin{bmatrix} y(0,t) \\ y'(0,t) \\ y(L,t) \\ y'(L,t) \end{bmatrix}$$

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 1 \\ \alpha_1 & 0 & \alpha_2 & 0 \\ \sinh(\alpha_1 L) & \cosh(\alpha_1 L) & \sin(\alpha_2 L) & \cos(\alpha_2 L) \\ \alpha_1 \cosh(\alpha_1 L) & \alpha_1 \sinh(\alpha_1 L) & \alpha_2 \cos(\alpha_2 L) & -\alpha_2 \sin(\alpha_2 L) \end{bmatrix}$$

$$[D] = \begin{bmatrix} \overline{D_1} \\ \overline{D_2} \\ \overline{D_3} \\ \overline{D_4} \end{bmatrix}$$

$\overline{D_1}$, $\overline{D_2}$, $\overline{D_3}$, $\overline{D_4}$ - constants

Equation (4) can be written considering moment, M, shear force, V, and the boundary conditions:

$$[F] = [B][D] \tag{4}$$

where,

$$[F] = \begin{bmatrix} M_{x=0} \\ V_{x=0} \\ M_{x=L} \\ V_{x=L} \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & EI\alpha_1^2 & 0 & -EI\alpha_2^2 \\ -(p\alpha_1 + EI\alpha_1^3) & 0 & -(p\alpha_2 - EI\alpha_2^3) & 0 \\ EI\alpha_1^2 \sinh(\alpha_1 L) & EI\alpha_1^2 \cosh(\alpha_1 L) & -EI\alpha_2^2 EI \sin(\alpha_2 L) & -EI\alpha_2^2 EI \cos(\alpha_2 L) \\ -(p\alpha_1 + EI\alpha_1^3)\cosh(\alpha_1 L) & -(p\alpha_1 + EI\alpha_1^3)\sinh(\alpha_1 L) & -(p\alpha_2 - EI\alpha_2^3)\cos(\alpha_2 L) & -(-p\alpha_2 + EI\alpha_2^3)\sin(\alpha_2 L) \end{bmatrix}$$

$$V = -\left(\frac{dM}{dx} + P\left[\frac{dy(x,t)}{dx}\right]\right)$$

$$M = EI\left[\frac{d^2y(x,t)}{dx^2}\right]$$

Equation (5) can be formed using (3) and (4) as follows:

$$[F] = [k]_L [Y] \tag{5}$$

$$[F] = [B][A]^{-1}[Y]$$

where,

$[k]_L$ –the dynamic stiffness matrix based on local coordinate system and subscript, L- local coordinate system.

The above stiffness matrix is defined based on the local coordinate system so that transformation matrix, [T] can be employed as follows to establish the stiffness matrix based on the global coordinate system:

$$[\overline{k_{Dyn}}]_G = [T]^T [\overline{k}]_L [T] \tag{6}$$

where, subscript G - the global coordinate system.

The dynamic stiffness matrix of the structure, $[\overline{K}]$, incorporating the influence of axial loads can be formed by assembling stiffness matrices of the elements considering compatibility of the nodes.

With the use of the dynamic stiffness matrix, $[\overline{K}]$, the equation of free vibration of a structure with the influence of the axial forces of elements can be represented as:

$$[\overline{K}]\{\overline{\phi}\} = 0 \tag{7}$$

where, $\{\overline{\phi}\}$ - modal vector of the structure.

Impact of the axial compressive force on the modal parameters can be examined through (7). However, it is not convenient to solve (7) to examine the modal parameters with the influence of the axial forces of a complex structural framing system with shear walls as shown in Example 02 (discussed in Section 3.2). Finite element package, ANSYS v.11.0 (ANSYS Inc., 2007) is thus modified considering the development presented above to capture effects of the applied axial compressive loads through the modal analysis. First example in Section 3.1 presents the validation of the modified FE program.

2.2 **Modal Flexibility of Element**

Modal Flexibility (MF) is indicative of the dynamic characteristics of a structure and incorporates both the modal vectors and natural frequencies. MF phenomenon is used to develop methodology proposed in this paper. This phenomenon is widely used in health or performance assessment of structures since it is accurate as well as convenient to apply to any structure. The Modal Flexibility of an element (element x), F_x of a structure can be obtained from (Adewuyi & Wu, 2010):

$$\quad {}_{r=1}\ \omega_r \qquad (8)$$

where,

x - the element considered,

r and n - the mode and total number of modes considered respectively, and

ϕ_{xr} -modal vector of element x for mode r

Note- Modal Flexibility is inversely proportional to the stiffness and ϕ_{xr} is a single entity at element x and hence F_x is a scalar. However, (8) is presented in the above format to be compatible with expressions in the previous publications.

Modal Flexibility (MF) for an element (element x) without the axial load (unloaded case) can be written as:

$$\left\lfloor \overline{{}_{r=1}}\ \omega_r \right\rfloor_U \qquad (9)$$

where, subscript U denotes the unloaded case.

As discussed previously, vibration characteristics of the element change due to the axial force. Modal Flexibility (MF) for element x with the axial load (loaded case) can be written as:

$$\left\lfloor \overline{{}_{r=1}}\ \omega_r \right\rfloor_L \qquad (10)$$

where, subscript L denoted the loaded case.

MF is a function of the stiffness matrix which changes with the axial force. In order to capture the influence of the axial force on the MF, the parameter, SI called the stiffness index is introduced through (11). This parameter is directly proportional to the stiffness reduction which occurs due to the axial load.

$$F_{xU} \qquad (11)$$

This stiffness Index (SI) parameter can be implemented for a structure as described below.

For the structure without axial loads, the modal parameters such as natural frequencies and mode shapes can be obtained numerically using the modified FE program (with axial load=0) and ambient measurements extracted from accelerometers deployed on the structure. Using these modal parameters, the Finite Element Model (FEM) can be validated and F_{xu} for element x can be calculated using (9) and reserved for future use. By applying known axial loads to both the FEM and the real structure, the above procedure can be repeated to improve the model validation.

In the next stage, the validated FEM of the structure is used to develop a database that relates the parameter SI to the axial deformation (AD). For a given axial load applied to the FEM of the structure, the modal parameters are determined using the modified FE program and the F_{XL} is calculated using (10) and then along with the F_{XU} determined earlier, SI for the particular case is calculated using (11). Axial deformation due to this axial force can also be obtained from static analysis. Repeating this procedure for a range of axial loads, a database for SI vs AD can be generated. Using the results from this database, graphs with the vertical axis representing the Stiffness Index (SI) and the horizontal axis representing the axial deformation (AD), can be plotted for each element x in the structure. In this research, (as will be seen later) the variation of SI vs. AD is linear. It is hence evident that, if SI is known (at any stage of loading or construction of the structure) the axial deformation AD can be obtained by applying either interpolation or extrapolation methods.

During the service life of a structure, the axial deformation (AD) of any element can be obtained from the SI vs AD graphs, if the SI is known. Under an unknown axial load on the real structure, the modal parameters amend due to the stiffness matrix change as addressed above and these parameters can be extracted from the deployed accelerometers and then used to calculate SI as described earlier. The axial deformation (AD) corresponding to the unknown axial load can then be obtained from the graphs of SI vs AD, already available.

3 VALIDATION AND ILLUSTRATIVE EXAMPLE.

First example is for validating the modified FE program used for the second example used to illustrate capability of the proposed method.

3.1 Validation- Example 1

Banerjee (1994, 2000) examined an axially loaded beam with cantilever end condition using different approaches. Results from all approaches confirmed their accuracy. The example used by these researchers was selected to study the accuracy of technique implemented in this paper. Fig. 03 shows a cross section of a beam element while Table 01 presents the material properties and other data used in the vibration analysis. More information of the selected element can be found in references (Banerjee 1994, 2000).

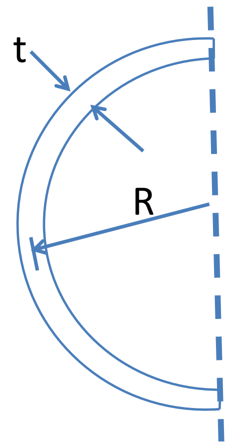


FIGURE 03: Cross section of the beam structure

TABLE 01: Material properties and other data in the vibration analysis

Beam Parameter	Numerical Value
R(mm)	24.5
t(mm)	4
L(mm)	820
m(Kg/m)	0.835
E(GPa)	68.9
G(GPa)	26.5

A finite element model of this element was developed using the modified FE program and was first subjected to an axial compressive force of 1790N and then to an axial tensile force of the same magnitude. These are the loads used in the previous publications (Banerjee, 1994, 2000). The natural frequencies of first three modes and corresponding mode shapes with and without axial loads are extracted from the analysis results and compared with results from the previous publications. Tab. 02, 03 & 04 show comparison of the results for the natural frequencies of the first three modes for three cases.

TABLE 02: Comparison of natural frequencies without axial load

Frequency Number	Natural Frequency(rad/s)		
	Axial force=0		
	Previous Publications	*Modal Analysis*	*Variations (%)*
1	391.70	390.40	0.33
2	816.00	815.00	0.12
3	1629.00	1625.00	0.25

TABLE 03: Comparison of natural frequencies with compressive axial load

Frequency Number	Natural Frequency(rad/s)		
	Axial force=1790N(compression)		
	Previous Publications	*Modal Analysis*	*Variations (%)*
1	405.80	404.00	0.44
2	826.70	826.00	0.08
3	1649.00	1650.00	0.06

TABLE 04: Comparison of natural frequencies with tensile axial load

Frequency Number	Natural Frequency(rad/s)		
	Axial force=-1790N(tension)		
	Previous Publications	*Modal Analysis*	*Variations (%)*
1	376.80	377.00	-0.05
2	805.10	804.00	0.14
3	1609.00	1608.00	0.06

It is evident from Tab. 02, 03 & 04 that variations between the present results and those from the previous publications are less than 1%. Additionally, the first three mode shapes obtained from the present analysis, where the first mode is bending and the next two modes are mostly torsional, compare well with the previous publications highlighting the accuracy of the modified FE program.

3.2 Example 2

As the last example, a 10 storey structural framing system with shear walls located at certain places is used to examine the impact of load migration due to the shear walls on the Stiffness Index (SI). Material properties used for elements in FEM are tabulated in Tab. 05. Sizes of columns and beams are 1x1m and 0.5 x0.5 m respectively while 0.5m thickness shear walls are employed in locations as shown in Fig. 04 Because of these shear walls; load migration occurs among the columns as in structural framing system with belt and outrigger systems. This example is hence enabling to study the capability of SI to capture the load migration. Floor height of the selected structure is 4m. Different axial compressive loads are applied on columns as tabulated in Table 06. These loads facilitate to simulate vertical elements of the structure subjected different loads from different tributary areas. Using the modified FE program, the analysis is performed with increasing all the applied loads by 0.25MN in order to develop several loading cases incorporating effects of axial deformations. The first two modes of vibration; both of which are bending modes and the corresponding frequencies are extracted from analysis to calculate SI(s) of columns. This is because higher modes do not impact significantly on SI. Stiffness Indexes, SI(s) of columns C1, C2, C3 and C4 shown in Fig. 04 at the certain floor levels are selected to examine their behaviour. SI(s) of columns in floor levels 2, 6 and 8 represent their behaviour at lower, middle and upper levels respectively, while SI(s) of columns in levels 4 and 8 represent the behaviour under load migration occurring due to shear walls located in these levels.

TABLE 05: Properties of elements

Property	Shear walls	Columns and beams
Young's Modulus/(GPa)	40	30
Poisson's Ratio	0.18	0.18
Density(Kg/m^3)	2400	2400

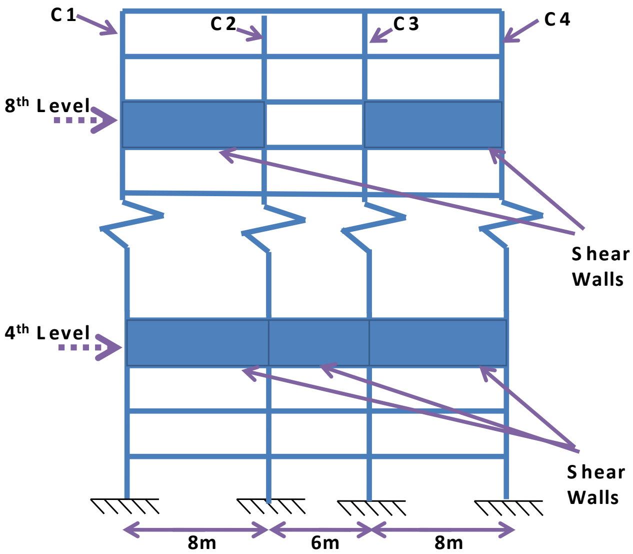


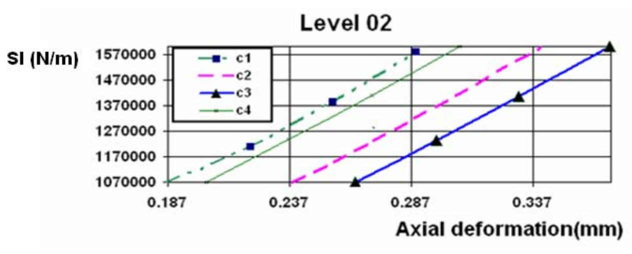
FIGURE 04: Structural framing system with shear walls.

TABLE 06: the applied axial compressive loads on columns initially

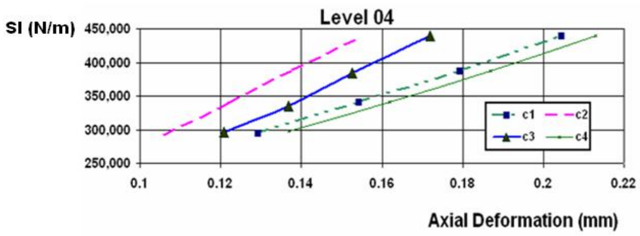
Floor Number	Column Force/ MN			
	C1	*C2*	*C3*	*C4*
1	1	1.5	2	1
2	1	1.5	2	1
3	2	3	3.5	2
4	1	1.5	2	1
5	1	1.5	2	1
6	1	1.5	2	1
7	2	2.5	3	2
8	1	1.5	2	1
9	1	1.5	2	1
10	1	1.5	2	1

Using the modified FEM program, separate modal analyses were performed incorporating effect of the applied axial compressive loads for each loading case. Stiffness Indexes, SI(s) are calculated for each column at the selected floor levels using the extracted modal parameters from the analysis results, while static analysis is used to calculate the axial deformations of the columns. Fig. 05 shows variation of SI(s) of the columns with their axial deformations at the selected floor levels.

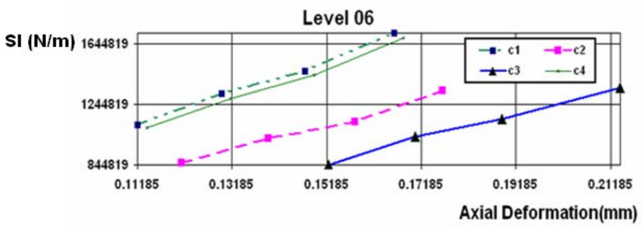
Fig. 05 (a), (b), (c) and (e) depict that SI of column C3 is lower than that of the other columns while SI of column C2 is low compared to columns C1 and C4. This is because column C3 is subjected to more axial compressive load than others and column C2 is subjected to higher axial load than that of columns C1 and C4 (see Tab. 06). However, SI of column C1 is higher compared to column C4 at 2nd and 6th levels and SI of column C4 is higher than that of column C1 at 10th level, though these two columns are subjected to equal axial loads (see Tab. 06). This is because load migration due to the shear walls impacts on axial deformations of columns C1 and C4.



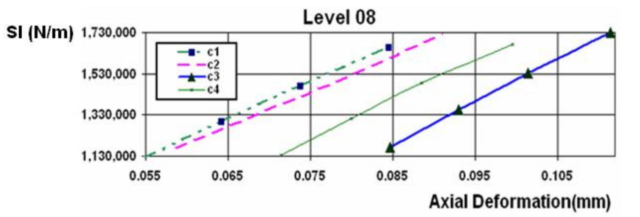
(a)



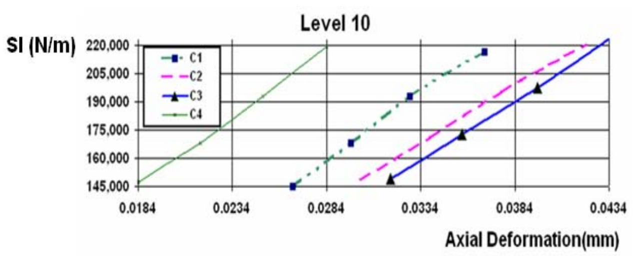
(b)



(c)



(d)



(e)

FIGURE 05: variation of SI(s) of the columns, (a)-2nd level, (b)- 4th level, (c)-6th level, (d)-8th level and (e)-10th level

In 4th level, axial load of column C3 migrates to columns C4 and C3 via the shear walls while axial load of column C2 migrates to column C1 only via the shear wall. Load migration from columns C3 to C4 is much higher compared to that of other two columns since axial load of column C3 is higher. Column C4 therefore acquires more loads than column C1. Consequently, Fig. 05(b) depicts that variation of SI of column C4 is low compared to the other columns and variation of SI of column C1 is lower than that of columns C2 and C3. This Figure also shows that SI of column C3 is lower in comparison to column C2 due to the fact that axial load and hence axial deformation of column C3 is more pronounced than the others According to shear wall configuration in 8th level (see Fig. 04), axial load of column C3 migrates to column C4 while axial load of column C2 migrates to column C1 only. However, axial load of column C3 is higher than other columns so that load migration from columns C3 to C4 is higher than other columns. Axial deformation of column C4 is thereby more pronounced than the other columns and hence SI of that column is low in comparison to column C3 as shown in Fig. 05(d). Fig. 05(d) also shows that variations of SI(s) of columns C1 and C2 are similar to columns C3 and C4 because of the load migrations. Nevertheless, the difference between columns C1 and C2 is low in comparison to the other two columns due to the fact that column C2 is subjected to low axial load than column C3 and hence load migration from columns C2 to C1 is low.

Moragaspitiya, Thambiratnam, Perera & Chan, (2010) reported that even though the axial deformations could be low, they impact significantly on the creep deformations which is a long term time dependent phenomenon, so that accurate quantification of the axial deformations are essential in order to provide adequate provision to mitigate adverse effects of differential axial deformations among vertical elements such as tilting horizontal floor plates, deformation of claddings facades, etc.

Fig. 05 indicate linear positive gradients of SI with axial deformation of elements in the structural system confirming that interpolation and extrapolation methods can be used to calculate the axial deformation due to unknown axial loads, when the SI is determined from vibration measurements. These linear graphs show that the elements deform axially in the linear elastic range and hence the stiffness reduces linearly. When axial deformation due to an unknown axial force is needed, modal vectors and natural frequencies can be obtained from the deployed accelerometers and the stiffness index, SI can be calculated. The axial deformation can then be obtained by using interpolation and extrapolation methods on the graph.

4 CONCLUSION

Axial deformations of columns are more pronounced in tall geometric complex structural framing systems so that measuring axial deformations using ambient measurements from gauges is conducted to mitigate adverse effects due to differential axial deformations. However, use of these gauges is expensive and inconvenient. Measuring vibration characteristics of structures to assess health/performance of structures is increasingly popular due to the fact that they can be acquired conveniently. This paper presents an innovative vibration based procedure to quantify the axial deformations of columns in structural framing systems through vibration based parameter called Stiffness Index (SI). Results of an illustrative example indicate that the proposed procedure with the parameter, SI has an ability to quantify axial deformation of elements in a structural framing system and capture the effects of the magnitudes of axial loads and the tributary area supported by the element as well as the load migration. The method proposed in this paper can be used to quantify axial deformation of an element of a complex structural framing system under gradual loadings using a non destructive test.

5 REFERENCE

Adewuyi, A.P., & Wu, Z.S. (2010). Modal macro-strain flexibility methods for damage localization in flexural structures using long-gage FBG sensors. *Structural Control and Health Monitoring,* Early View. DOI: 10.1002/stc.377

ANSYS Inc. (2007). ANSYS Workbench v.11.0 [Software]. Canonsburg, PA: Ansys Inc.

Banerjee, J.R. (2000). Explicit modal analysis of an axially loaded Timoshenko beam with bending-torsion coupling. *Journal of Applied Mechanics, 67*(2), 307-313.

Banerjee, J.R., & Williams, F.W. (1994). Coupled bending-torsional dynamic stiffness matrix of an axially loaded Timoshenko beam element. *International Journal of Solids and Structures, 31*(6), 749-762.

Brownjohn, J.M.W., Pan, T.C., & Deng, X.Y. (2000). Macro-updating of finite element modeling for core systems of tall buildings. In *Proceedings of the 14th Engineering Mechanics Conference*. Austin, TX: American Society of Civil Engineers.

Carreira, D.J., & Poulos, T.D. (2007). Designing for effects of creep and shrinkage in high-rise concrete buildings. *ACI Special Publication*, *SP-246-7*, 107-131.

Dubai Future Projects. (2009). Retrieved January 1, 2009, from http://www.realtyna.com/dubai_real_estate/dubai-future-projects.html

Ellis, B.R., & Ji, T. (1996). Dynamic testing and numerical modeling of the Cardington steel framed building from construction to completion. *The Structural Engineer, 74*(11/4), 86-192.

Moragaspitiya, P., Thambiratnam, D.P., Perera, N., & Chan, T.H.T. (2010). A numerical method to quantify differential axial shortening in concrete buildings. *Journal of Engineering Structures, 32*(8), 2310-2317.